On The Human Eye

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I. Recap of Fourier Theory

Monsieur J. Fourier

His transformation relation

$$\int_{-\infty}^{\infty} \exp(-i\omega t) f(t) dt = S(\omega)$$

Time Domain

Frequency Domain

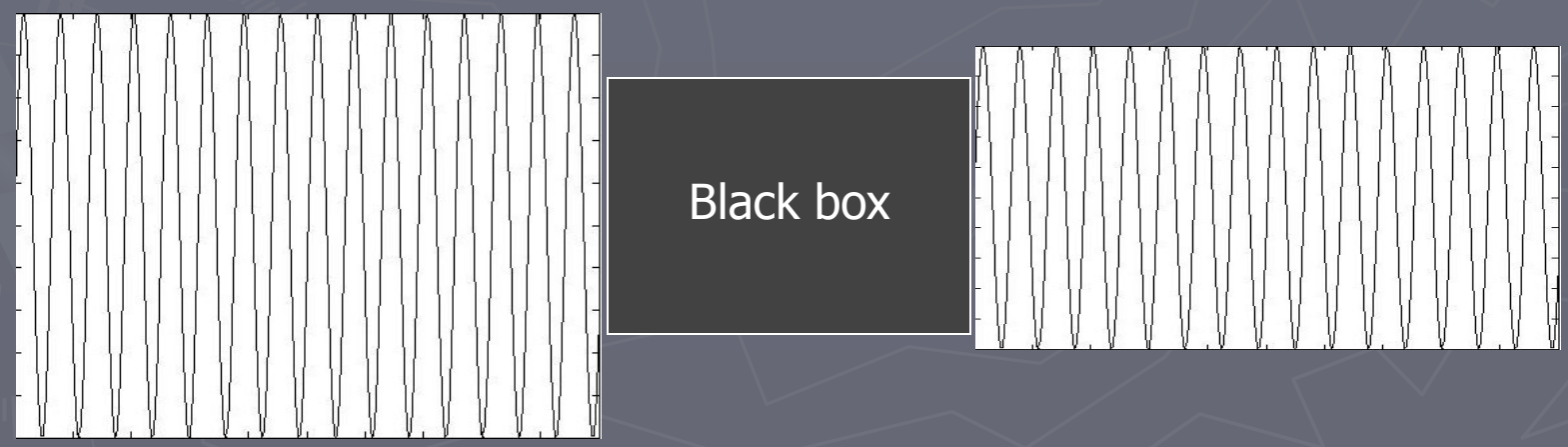
$$\int_{-\infty}^{\infty} \exp(-i\vec{k}.\vec{r}) f(\vec{k}) d\vec{k} = S(\vec{r})$$

Momentum Space

Geometric Space

Has to do with exploring nature

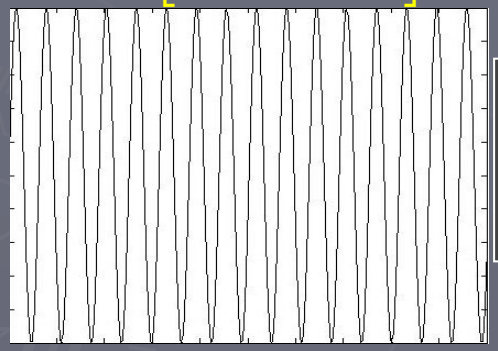
We learn about the “b box” by periodically perturbing it:



Eigenfunction of Linear time-invariant systems

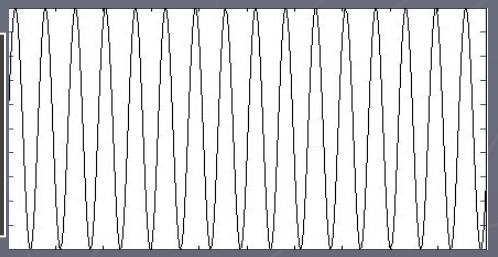
$$f_{black_box}\left\{\exp\left[i\left(\vec{k}.\vec{r}-\omega.t\right)\right]\right\}=S(\omega).\exp\left[i\left(\vec{k}.\vec{r}-\omega.t\right)\right]$$

$\exp\left[i\left(\vec{k}.\vec{r}-\omega.t\right)\right]$



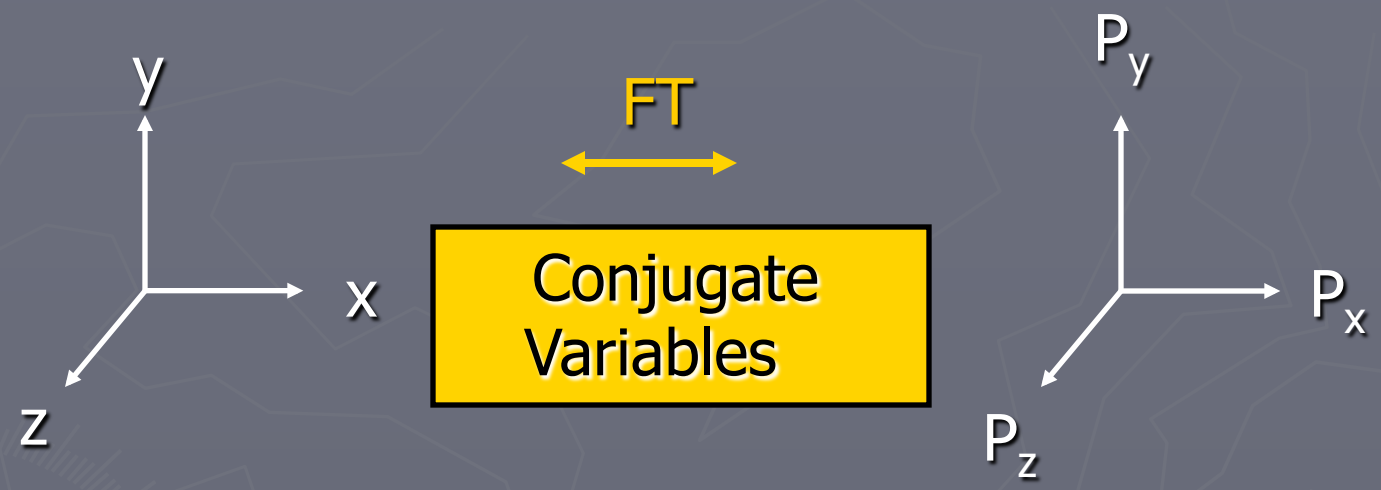
Black box

$S(\omega).\exp\left[i\left(\vec{k}.\vec{r}-\omega.t\right)\right]$



$S(\omega)$ = Eigenvalues = Spectrum (w) =Frequency Response Function

Two views of same object



Quoted by Prof. Richard Ernst

“ The most fruitful developments have happened whenever two different kinds of thinking were meeting”

- Werner Heisenberg

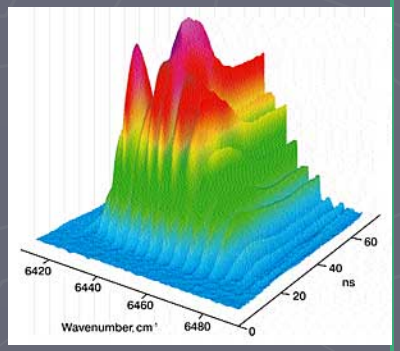
What else…

Generalize the Eigenfunctions

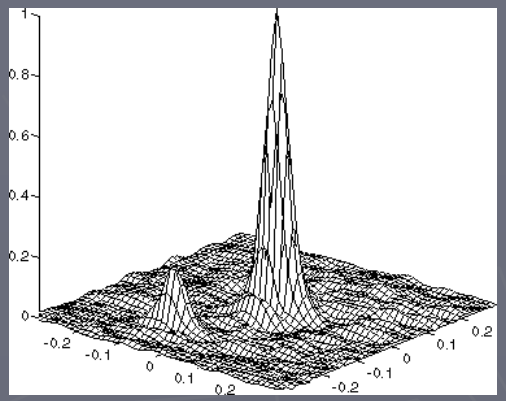
$$\exp\left\{i(\vec{k}.\vec{r} - wt)\right\}$$

The world of Spectroscopy is quite rich. Integrating over:

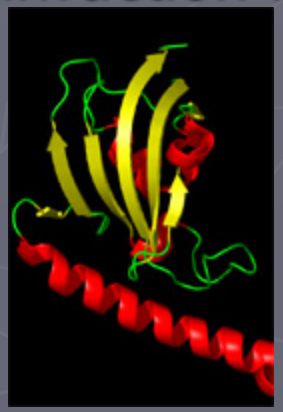
Time (t): Time domain Spectroscopy

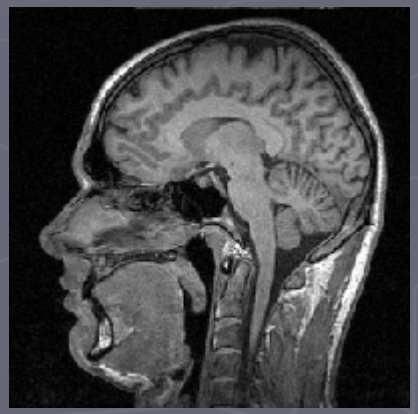


Space (r): Interferometry



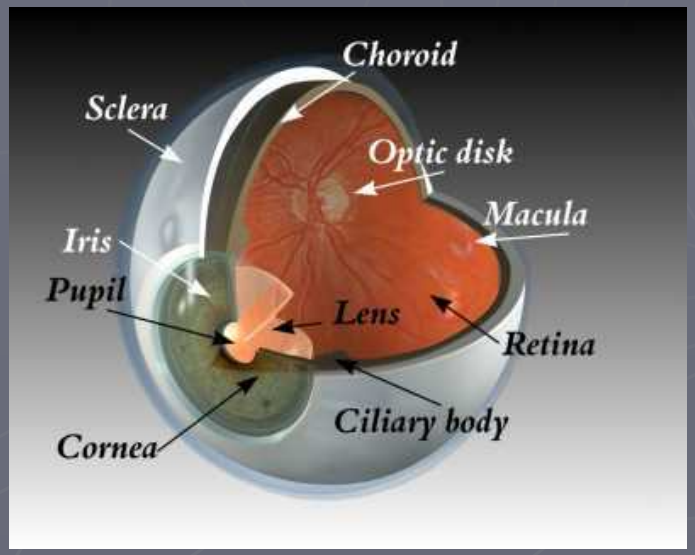
Spatial frequency (k): Diffraction imaging





II. The Eye: quick overview

- Provides 80% of the information.
- 97% of the light directed at the pupil does not result in perception. Scattering and reflection.



For tons of info: See C. Oyster

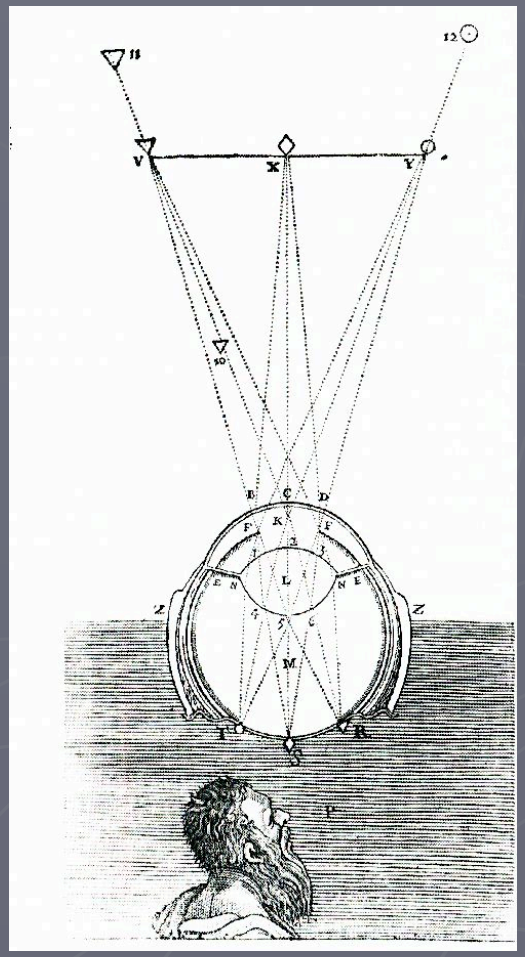
The eye : refracting components

- RETINA: first encounter of the brain with light.
- If there is NOT and EYE

→ Point Spread Function on a film: Like Holographic domain. Like SNAILS, or OCELLI !

- *EVOLUTION'S SOLUTION*
- The lens and cornea:

  Emulate and propagate the sense of touch to distances.



Descartes' depiction of the Eye's functioning

Fourier relation for images

$$F\{g\} = \iint_{\circ 2} g(x,y)\exp\left[-j2\pi\left(f_X x + f_Y y\right)\right]dxdy$$

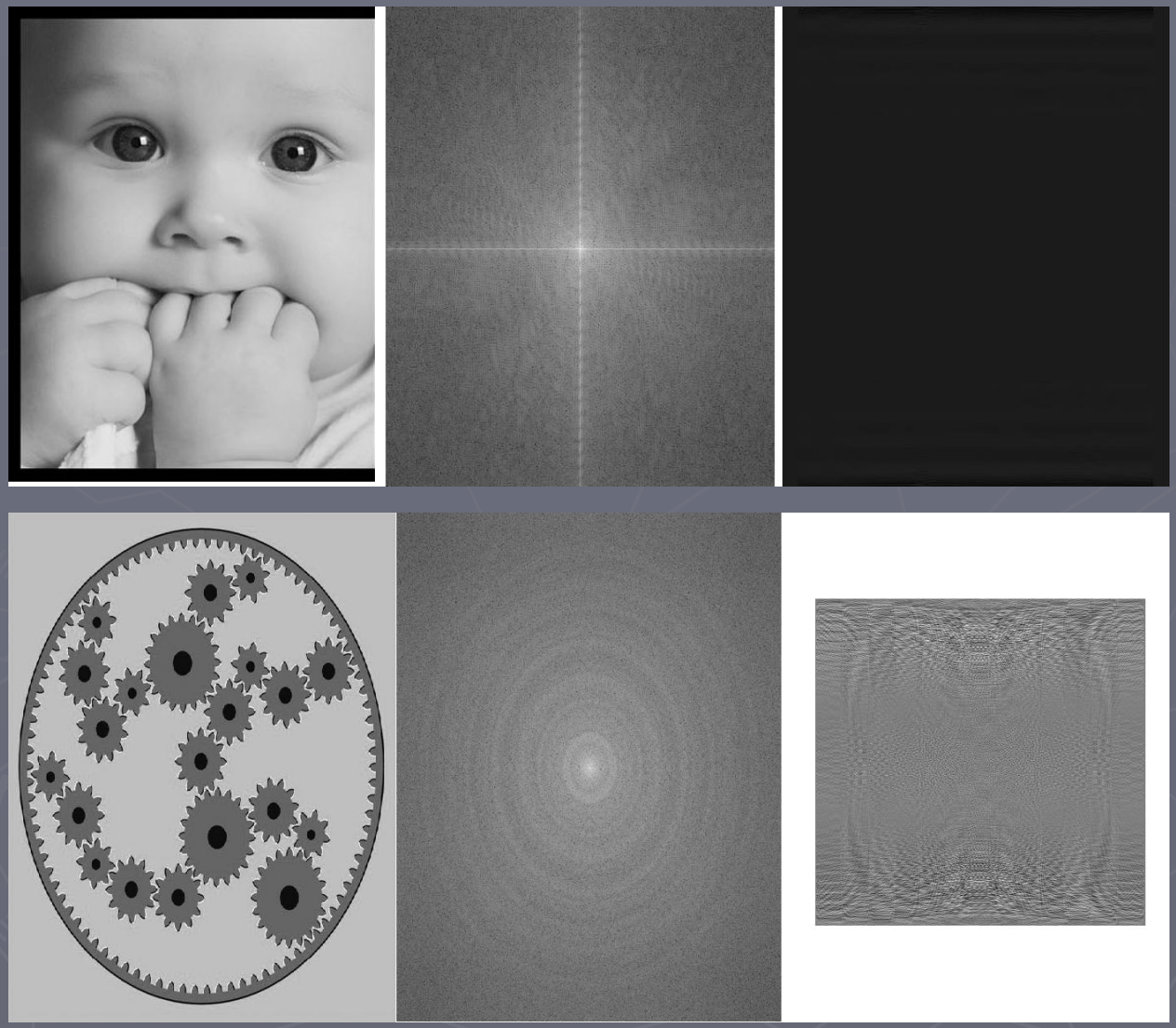
$$F^{-1}\{G\} = \iint_{\circ 2} G(f_X, f_Y)\exp\left[j2\pi\left(f_X x + f_Y y\right)\right]df_X df_Y$$

CODE

Fourier Transformation of Images

Way of compressing data (like JPEG)

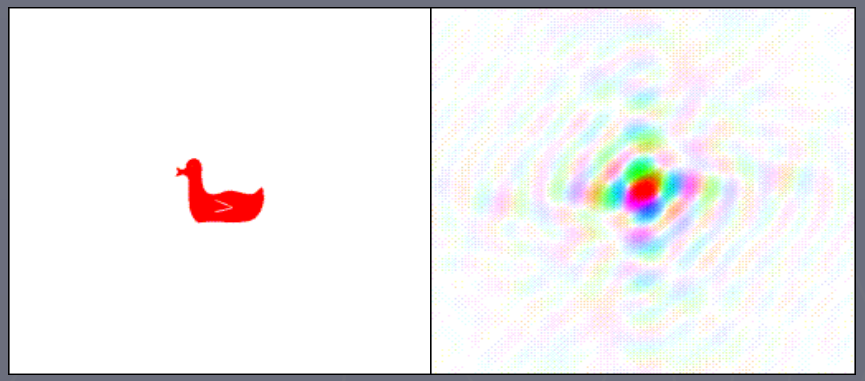
III. Illusions



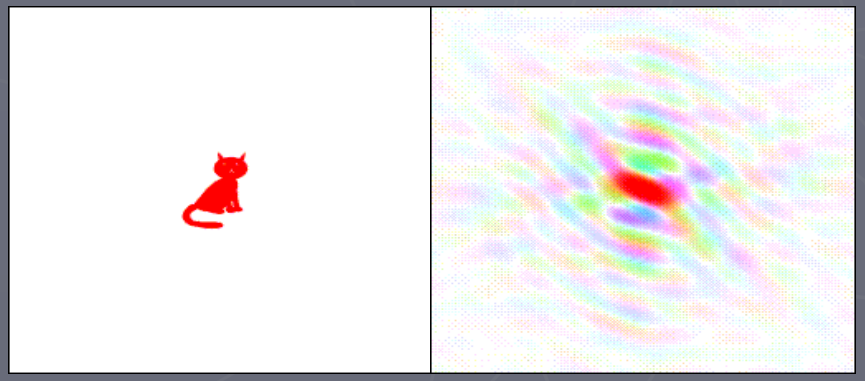
The image F. T. Phases

Importance of phases

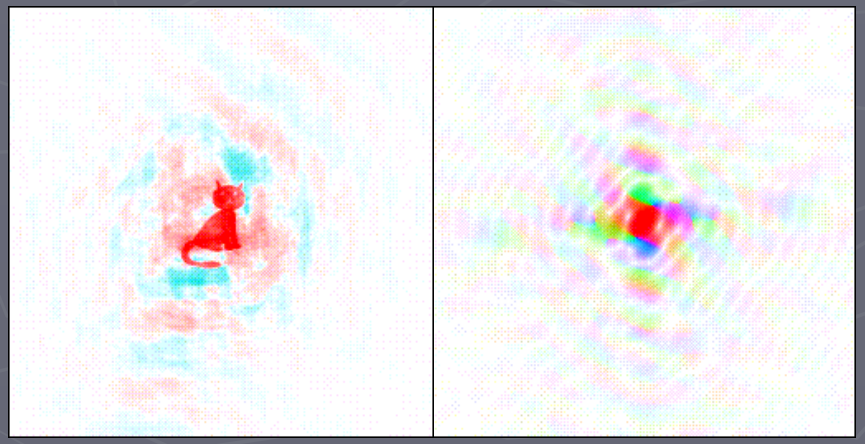
Dock and the Fourier Dock



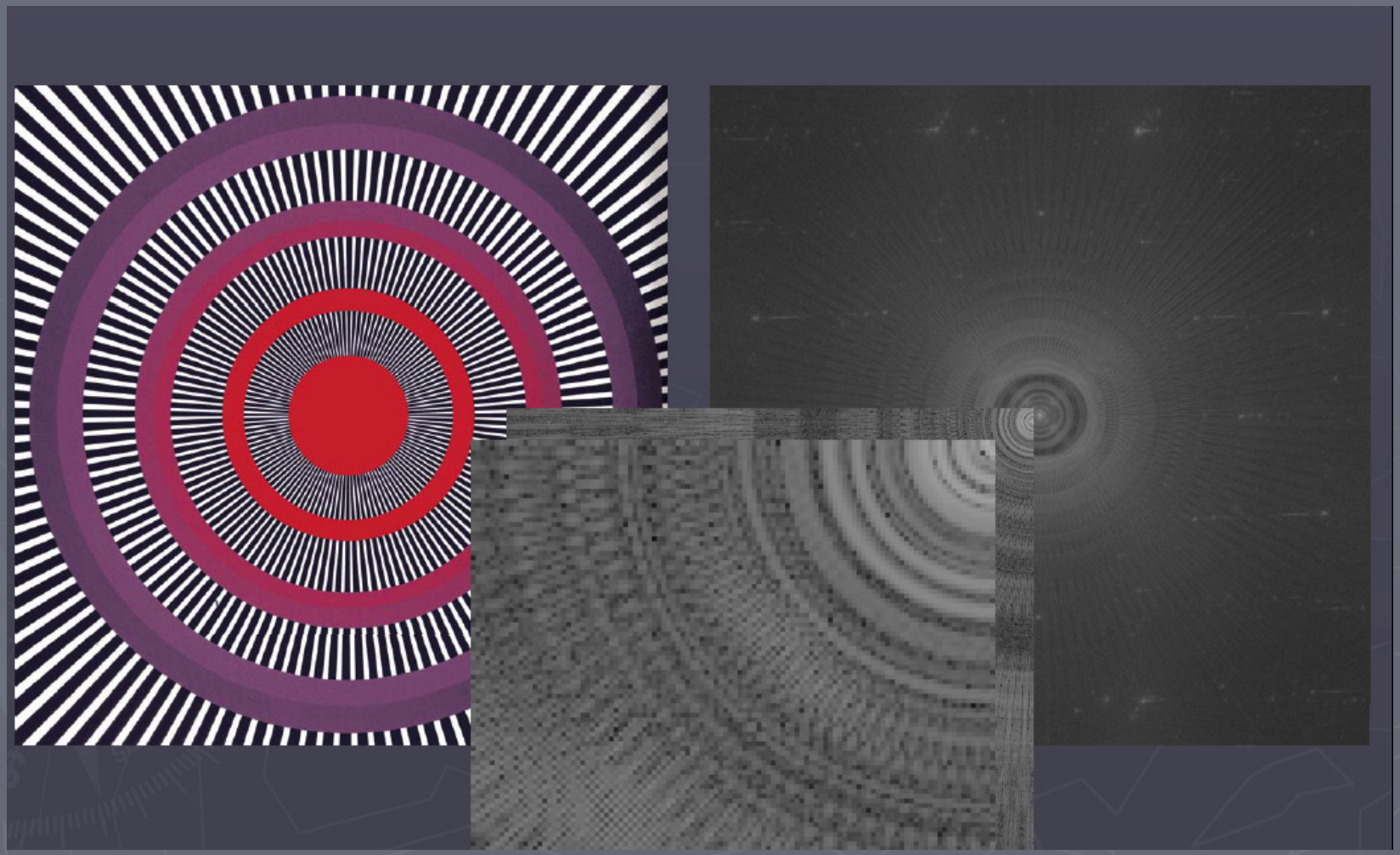
Cat and the Fourier Cat



Reconstruction from phase of the cat and the amplitude of dock

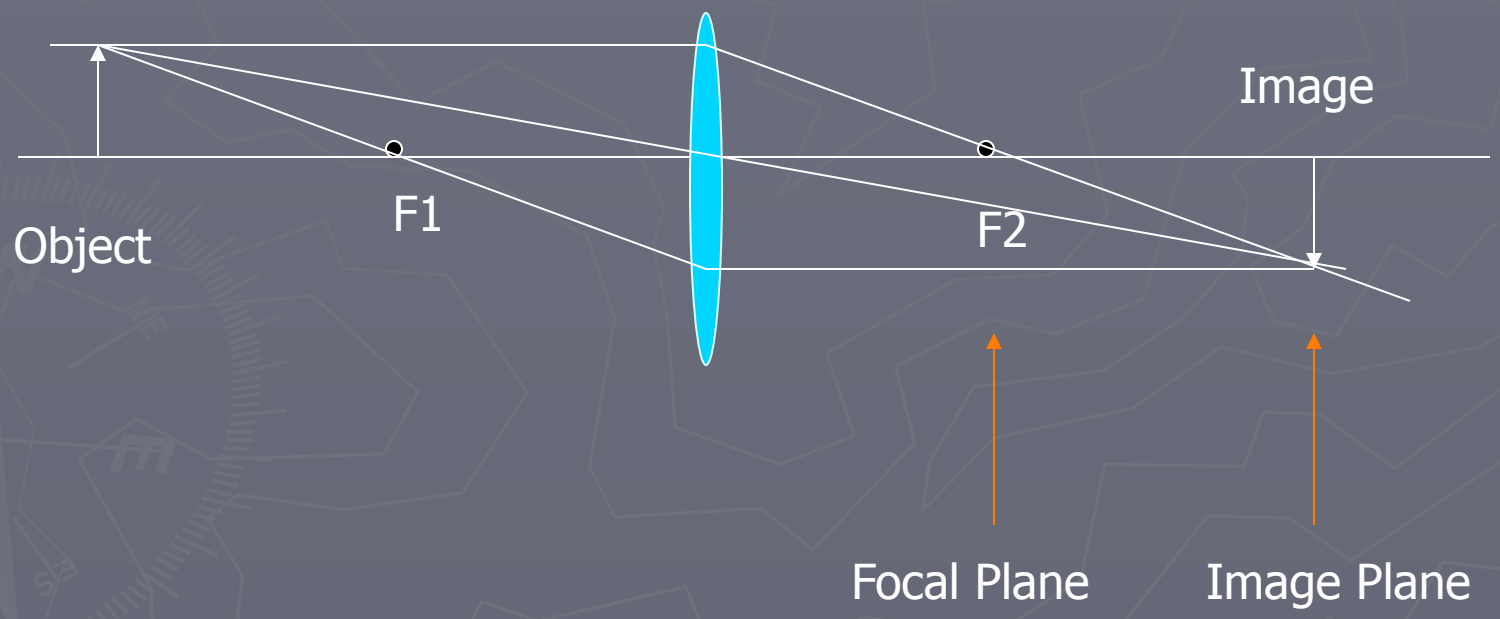


Could it be that the complexity of the Fourier Space is THE reason for the instability (-> Illusions) of the visual system?



IV. Basic theoretical tools

- Geometrical Optics



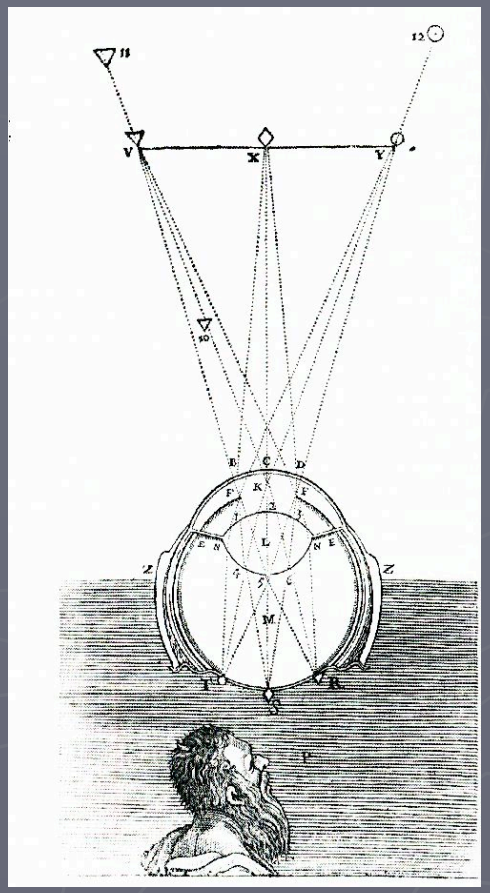
Retina Image Quality

- Descartes' classical model of the human eye.

Guided by past experiments and common sense:

- We believe in PLANE OF MAXIMUM INFORMATION behind the lens.

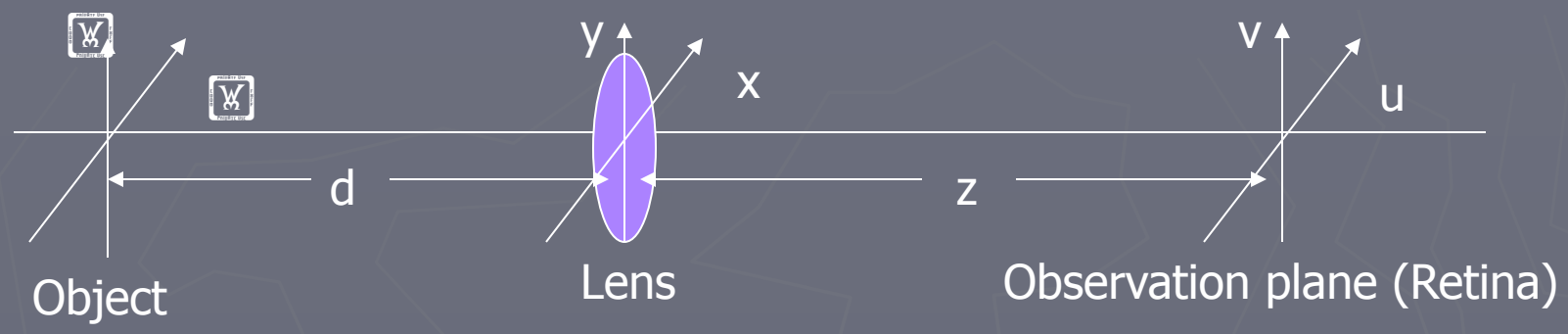
- Minimizing THE SHANNON ENTROPY → maximize information.



Descartes' model

V. Theoretical Approach

Consider the following schematics:



Integral that determines the image quality behind the lens.

$$U(u,v,z) = \frac{e^{jkz}}{j\lambda z} e^{j\frac{k}{2z}(u^2+v^2)} \times \iint_{\circ\,2} U_l(x,y) P(x,y) e^{j\frac{k}{2}(x^2+y^2)(\frac{1}{z}-\frac{1}{f})} e^{-j\frac{2\pi}{\lambda z}(xu+yv)} dxdy$$

$$U_l(x,y) = F^{-1}\left\{ F_0(f_x,f_y) e^{-j\pi d\lambda(f_x^2+f_y^2)} \right\}$$

P(x,y): Pupil function
$U_l(x,y)$: Input to the lens. That is the object after free propagation distance d.
F and F^{-1} refer to Fourier and Inverse Fourier transformations.
$F_0(fx,fy)$: Fourier transformation of the input 'object'

Minimizing Shannon's Entropy

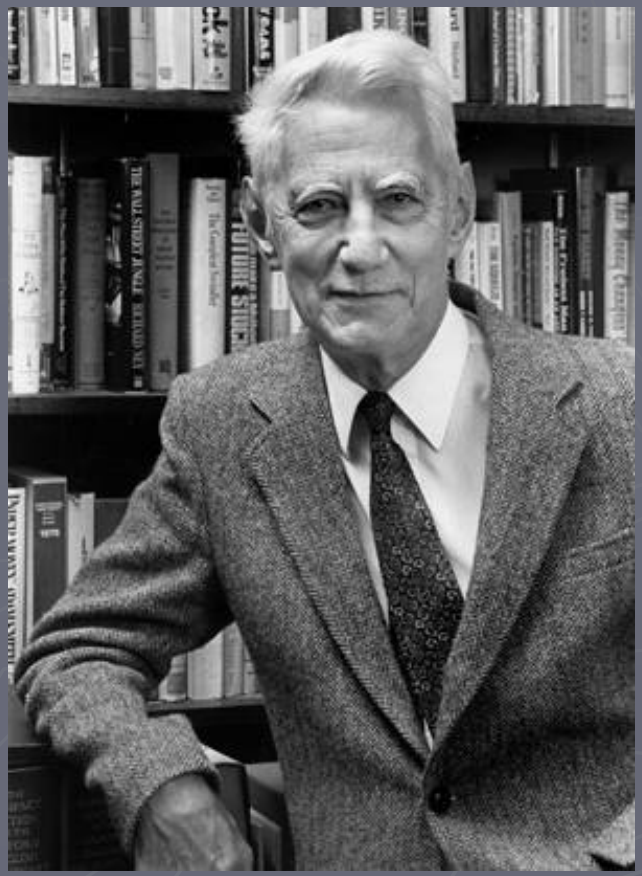
Definition: The concept of entropy in information theory describes how much randomness (or, alternatively, 'uncertainty') there is in a signal.

$$H(x) = -\sum_{i=1}^{n} p(i) \log_2 p(i).$$

x: an event

i: all possible outcomes of x

Information and entropy relation: $\therefore H \propto \frac{1}{I}$

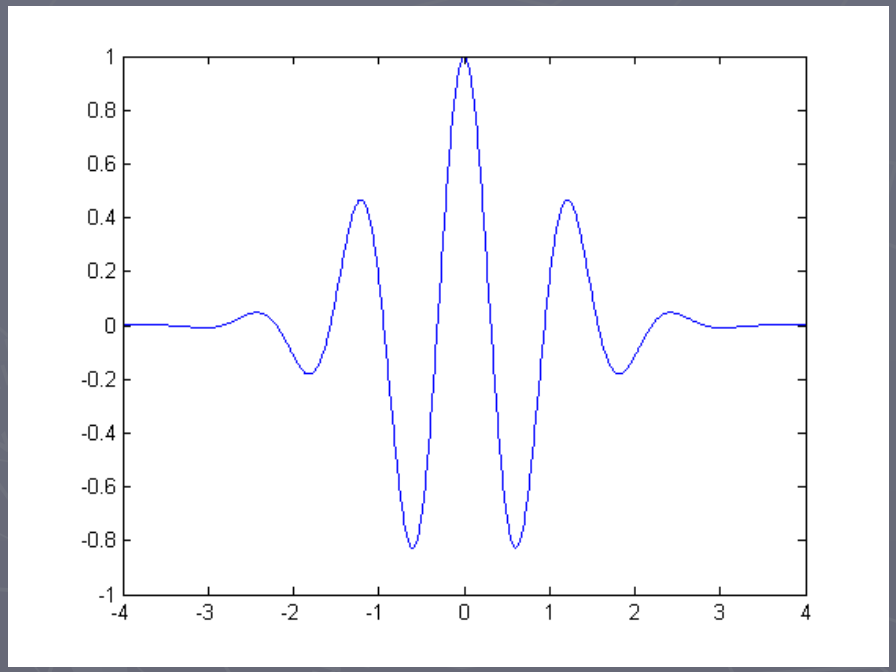


Claude Elwood Shannon
(1916-2001)

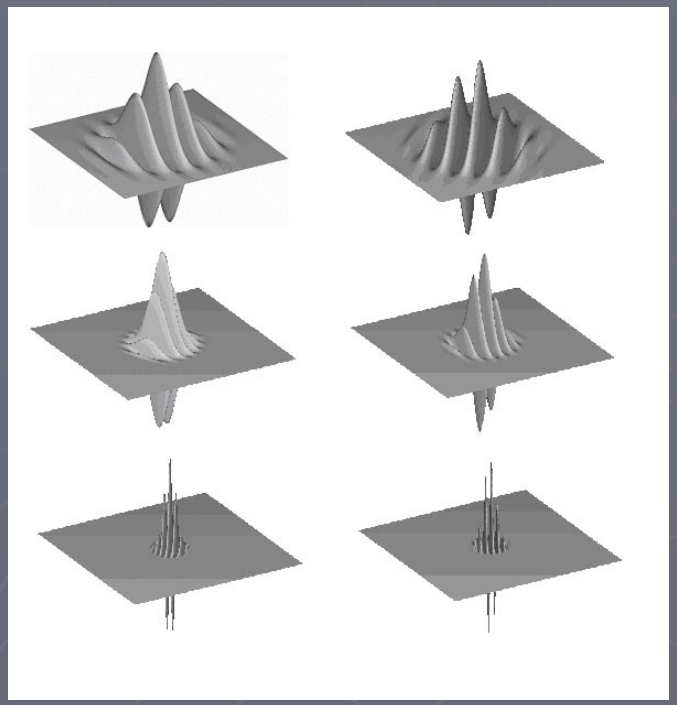
Wavelets

- The world between the Fourier and Real space.
- One has to respect the geometry of the problem
- For example the finiteness or the info content of the signal.

Typical Wavelet Basis

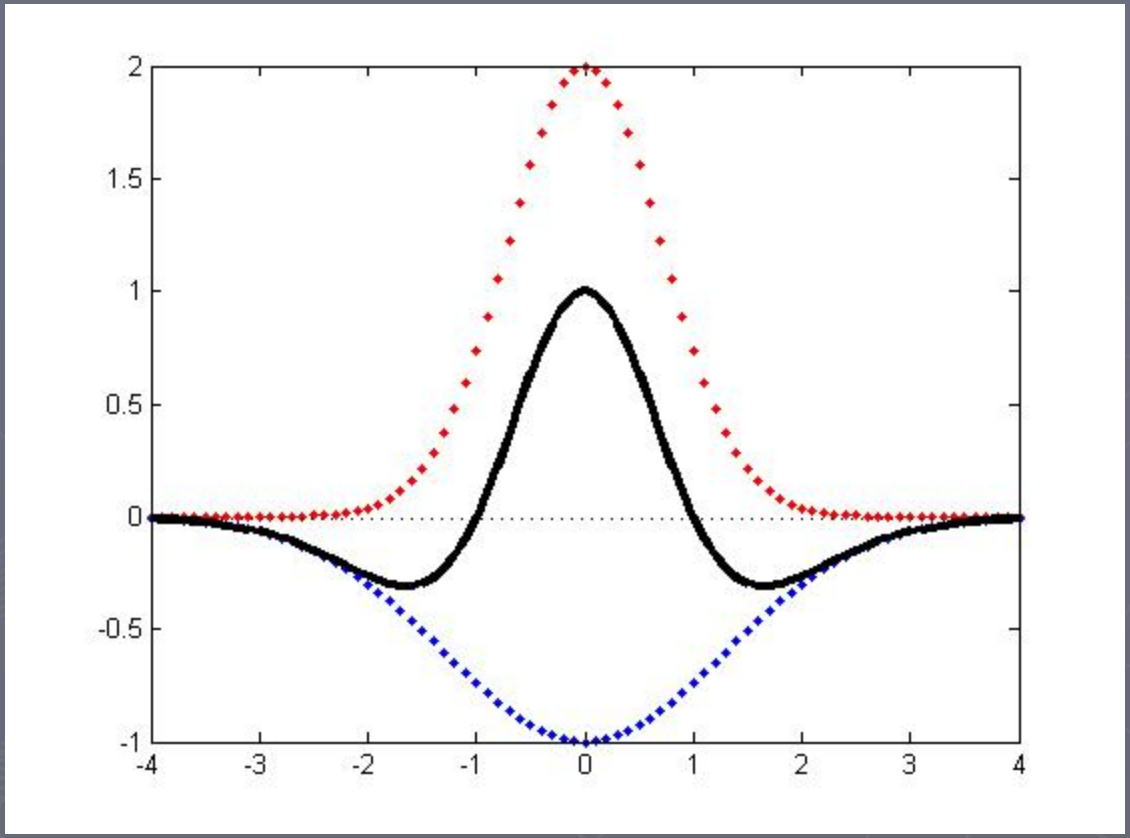


Mother Wavelets



2D dimensional Wavelets

Can we predict Center Surround Information Theoretically??!



Difference of Gaussian representing the CENTER-SURROUND (e.g. for Ganglion cells).

Acknowledgements

- Reinhard Nesper
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- Richard Ernst

All of you !!